

Problem Set 6
Optical Waveguides and Fibers (OWF)
will be discussed in the tutorial on December 9, 2015

Exercise 1: Modes at a dielectric-metal interface: Surface plasmon polaritons.

The real part of the dielectric constant can sometimes be negative, like in the case of metals at frequencies smaller than the plasma frequency. In this case it is possible to have guided modes that are confined to the boundary between the metal and a dielectric. The aim of this exercise is to describe these modes (called “surface plasmon polaritons”) starting from the slab waveguide description by letting the thickness of the core tend to zero, see Fig. 1. Consider a slab waveguide defined by three layers having relative

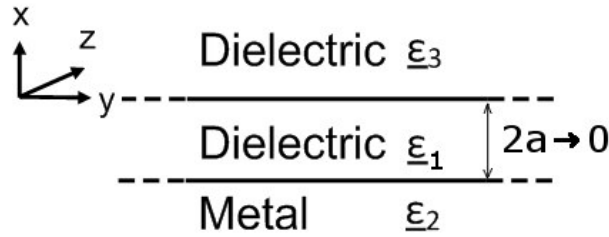


Figure 1: Metal-dielectric interface.

dielectric constants ϵ_3 (top cladding), ϵ_1 (core) and ϵ_2 (bottom cladding). The layers are oriented parallel to the yz -plane and the thickness of the core layer is $2a$, as drawn in Fig. 1. For the guided fundamental TM-mode of an asymmetric slab waveguide, the propagation constant β is determined by the following equation:

$$u = \frac{1}{2} \arctan\left(\frac{\epsilon_1 w}{\epsilon_2 u}\right) + \frac{1}{2} \arctan\left(\frac{\epsilon_1 w'}{\epsilon_3 u}\right), \quad (1)$$

with the normalized parameters defined as

$$\begin{aligned} u &= a \cdot k_{1x} = a \sqrt{\epsilon_1 k_0^2 - \beta^2} \\ w &= a \cdot k_2^{(i)} = a \sqrt{\beta^2 - \epsilon_2 k_0^2} \\ w' &= a \cdot k_3^{(i)} = a \sqrt{\beta^2 - \epsilon_3 k_0^2}. \end{aligned}$$

- a) Let the thickness a tend to zero. Show that the following condition must be satisfied:

$$\frac{w}{\epsilon_2} = -\frac{w'}{\epsilon_3} \quad (2)$$

- b) For a guided mode, w and w' must be real and positive. Assume further that both ϵ_2 and ϵ_3 are real. From Eq. (2) we find that ϵ_2 and ϵ_3 must have opposite sign in order to obtain a guided mode. Calculate the propagation constant β from Eq. (2). Show that a propagating mode with real β exists only if $\epsilon_2 < -\epsilon_3$.
- c) Identify all non-zero field components of the slab waveguide TM-mode along with the boundary conditions that these fields must fulfill at the metal-dielectric interface. All components must decay evanescently into the top and the bottom cladding. Sketch qualitatively all non-vanishing field components for $\epsilon_3 = 1$, $\epsilon_2 = -4$.

Questions and Comments:

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